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A Manually Retargeted Automatic Landing System for the Lunar Module (LM)

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During the final few minutes and several miles of the Apollo lunar landing, the commander of the Lunar Module (LM) can manually retarget the automatic landing system. At the beginning of this phase, the approach phase, the automatic landing system is targeted to a preselected landing site. The LM's computer identifies the landing site to which it is currently targeted by orienting the LM about the thrust axis such that the site appears to coincide with a window-fixed reticle, and displays a number indicating the point on this reticle through which to look. The commander identifies the current site and selects the desired landing site as a previously landed spacecraft, a natural landmark, or an appropriate unmarked site. Estimating the angular error between the current and desired landing sites, the commander steers to null this error by repetitively manipulating a hand controller. The controller signals go only to the automatic system which counts them, moves the current site through a proportional angular increment (as seen by the commander), and computes the new target parameters and the number to display indicating the new current site. By steering until the point on the reticle currently indicated by the computer coincides with the desired site, the LM commander can converge on any landing site he may select.

Introduction

DURING the final 2 or 3 min and 4 to 10 miles of his approach to the moon (Fig. 1) the commander of the Lunar Module (LM) must visually survey the area that he approaches, choose a landing site within this area, and steer the LM to that landing site. This paper describes the system proposed for the approach phase of the Apollo lunar landing.

Prior to the approach phase, there is a completely automatic braking phase which takes the LM from lunar orbit to those target conditions which properly set up the approach phase. After the approach phase, there is a transition to hover and a vertical descent to touchdown. The braking phase and the transition to hover use the same guidance equations as the approach phase. The hover and vertical descent use a simple velocity-error-nulling technique. In this paper, attention is focused primarily on the approach phase, the only phase in which the essential features described here are available.

The essence of the approach phase guidance system is that the LM commander can manually steer the LM to the selected landing site, yet the trajectory he flies is produced by an automatic guidance system. This hybrid combination allows very nearly continuous manual steering in the form of commands which incrementally shift the point on the surface at which the LM will land, yet the automatic system provides very nearly the most efficient trajectory to the selected landing site consistent with certain constraints. Aside from the obvious desire of any man to control his craft, and aside from the general flexibility this capability provides, the system fulfills the following specific objectives:

1) The LM commander must assess the safety of his trajectory. He must be assured impact is not imminent

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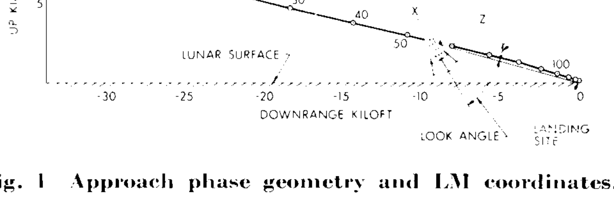

Fig. 1 Approach phase geometry and LM coordinates.

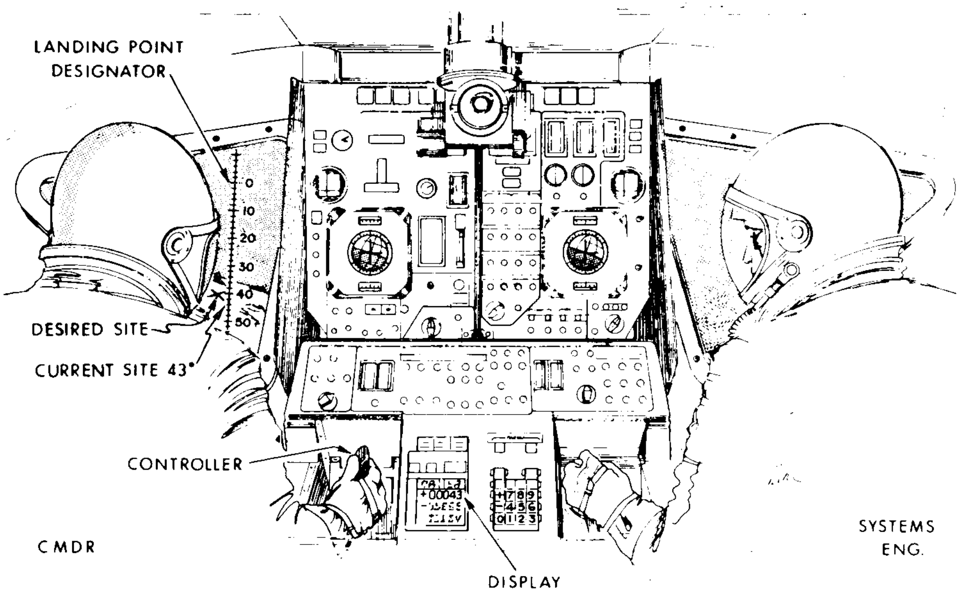

Fig. 2 Components of the steerable landing system.

either because of equipment failure, which could result in a dangerous relationship between altitude and altitude rate, or because of some gross terrain obstruction along his course to the landing site. Although, by prediction, it will be very difficult to judge altitude, it should not be difficult to judge the relationship between altitude and altitude rate (specifically the time to impact at the current altitude rate). The landing system provides the visibility of the surface required to make such assessments and the capability to steer clear of terrain obstructions.

2) It may be required to land at a specific site chosen before launch. Possible targets include a previously landed spacecraft and natural landmarks of special interest such as specific craters. The guidance to any of the possible landing targets is to be by visual identificiation and manual steering.

3) Near the end of the descent it probably will be necessary to change the direction of the trajectory in order to approach the landing site from a direction of more favorable lighting. Certain constraints, the discussion of which is beyond the scope of this paper, require the LM to begin its descent in a direction such that the sun is within a few degrees of the plane of the descent trajectory and directly behind the LM. With this lighting geometry, much more detail is visible 20° or 30° to the side of the approach trajectory. The steerable landing system provides a convenient method to change the direction of the trajectory near the end, where the cost in fuel of making such a change is small, and to converge on a site judged adequate through detailed scrutiny and favorable lighting.

Functional Description

Figure 1 is an elevation view of the approach phase of a typical landing trajectory. The trajectory is shallow (the depression angle is small) because shallow trajectories are

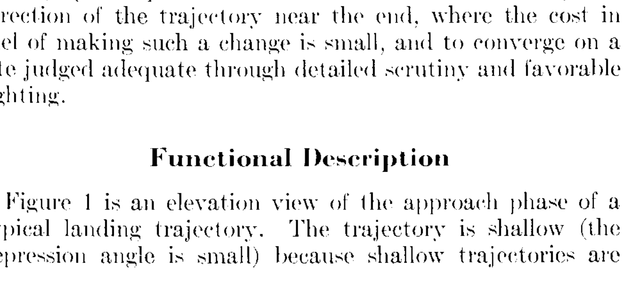

Fig. 3 Approach phase guidance.

more efficient fuelwise. The optimum trajectory would approach the landing site nearly horizontally. However, the shallower the trajectory, the less the detail the crew can discern and the less apparent are any shadows cast from the rear by the sun. Thus, the depression angle is a compromise between fuel cost and visibility. Note that the shaping of the trajectory produces an increasing depression angle as the landing site is approached, in the region where visibility is of prime concern. Figure 2 shows a view through the commander's window and the key elements in the manual steering system, namely, the computer display, the systems engineer, the commander, the landing point designator (LPD), and the controller. Figure 3 pertains to the following description of how these elements work together to steer the LM to the selected landing site.

At the beginning of the approach phase the LM assumes an attitude such that the surface and the landing site are visible, and the commander visually scans the moon for the desired landing site. He will recognize it either by a marker placed by a previously landed spacecraft, as a visually identifiable landmark, or as an appropriate though unmarked site. Meanwhile, the computer orients the LM such that the thrust points in the direction required for reaching the current site (that resulting from previous steering, if any, where the LM will land if there is no further steering) and uses the remaining degree of freedom (about the thrust axis) to keep the LPD superimposed on the current site. (Figure 2 does not show that there are two LPD scales, one inside the window, one outside, to allow the commander to register his eye.) The computer also repetitively calculates and displays the complement of the look angle (Fig. 1) of the current site. The systems engineer repetitively reads the angle from the display and repeats it to the commander. The commander identifies the current site by sighting through the LPD and observes the angular error, if any, between the current site and the desired site. If the angular error is significant, he manipulates the controller to cause the computer to redefine the current site closer to the desired site. Through repetition of the total process, the commander literally steers the current landing site into coincidence with the desired site.

Meanwhile, the computer repetitively plans the trajectory to the current site. This consists of adjusting the time remaining in the phase such as to provide suitable visibility, and adjusting the terminal conditions such as to keep the position, velocity, and acceleration vectors, the planned landing site vector, and the LM Z and X axes all coplanar at the approach phase terminus (typically 117-ft altitude). From among the class of trajectories at the disposal of the computer, it produces that trajectory which arrives at the phase terminus in very nearly the shortest time, and therefore uses very nearly the least fuel, consistent with the constraints on visibility, etc. The commander may steer until about 15 sec prior to the end of the phase, when the computer will stop accepting steering commands.

Each manipulation of the controller causes the computer to change the landing site by a fixed angular increment, as seen by the commander. This angular increment is resolved in body coordinates. Thus, the effect of a steering command appears the same to the commander regardless of the LM attitude or the range to the current site. Because the magnitude of the angular increment is the same at all points along the trajectory, the response of the spacecraft to an incremental command is very nearly the same at any point. It is not possible to introduce inadvertently a gross site redesignation which could result in violent response in thrust magnitude and spacecraft attitude.

Design Objectives for Guidance Logic, Attitude Control Logic, and Trajectories

The list of design objectives in this section is an arbitrary set chosen to test the proposition that given a reasonable set of specifications in this format, the guidance and attitude control logic permit trajectories to be designed which meet the specifications. None of the numbers has official sanction.

Figure 4 shows a landing footprint that was chosen as an arbitrary objective for steering capability. This footprint shows that if a landing maneuver were initiated at some altitude H_0 (roughly 8000 ft for the sample trajectory described at the end of the paper) and at a ground range of 4 H_0 from the unredesignated landing site, then the objective is to be able to reach any landing point in an ellipse extending 3 H_0 in front of the unredesignated site and $\frac{5}{8}$ H_0 to either side. Any trajectory produced by redesignation at this initial point and landing anywhere in the ellipse should meet all the objectives. Trajectories can be flown which land outside of the footprint, but they may fail to meet one or more of the objectives.

It may be noticed from Fig. 4 that the entire landing footprint lies forward from the unredesignated landing site. This permits the preplanned trajectory (unredesignated) to use minimum fuel consistent with visibility constraints; i.e., on the preplanned trajectory, visibility is barely within the required limits. Forward steering would improve visibility; backward steering would degrade visibility beyond the visibility limits.

1) Characteristic velocity objective: The characteristic velocity shall be near the minimum consistent with meeting the other objectives.

2) Steering objectives: a) It shall be possible to select and steer to a new landing site anywhere within the landing footprint of Fig. 4 with the resulting trajectory meeting all of the design objectives, providing the gross targeting correction is completed in the first 15 sec of the approach phase. b) It shall be possible to select and steer to a new landing site at any time during the approach phase, but it shall be the responsibility of the commander to use proper judgement in the magnitude and rate of application of the site redesignation such as to maintain adequate visibility and attitude excursion limits. c) It shall be possible to steer to convergence upon a previously selected site throughout the approach phase as long as the site is visible.

3) Visibility objectives: a) The landing site shall lie at least 10° above the bottom edge of the window (the look angle shall be at least 35°) from the beginning of the approach phase, for a minimum of 75 sec, and until it recedes from view at the bottom of the window just prior to phase terminus. b) The landing site shall not recede from view at greater than 300-ft slant range. c) No site redesignation within the conditions of objective 2a shall cause the landing site to disappear. d) The depression angle (Fig. 1) shall be greater than 15° for at least 15 sec prior to phase terminus.

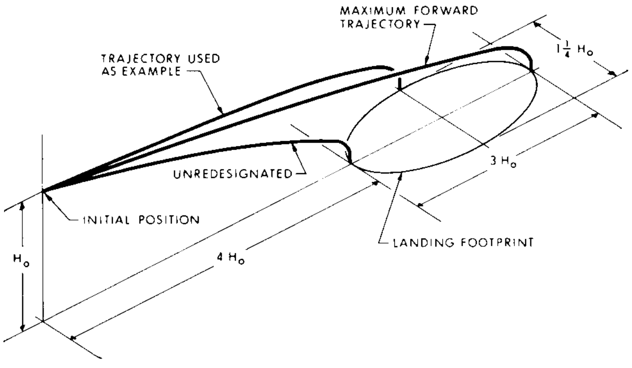

Fig. 4 Landing footprint—an objective for steering capability.

4) Attitude objectives: a) The attitude shall be controlled to keep the landing site in the LM symmetry plane at all times when thrust pointing requirements permit site visibility and to minimize the angle between the normal to the trajectory plane and the LM pitch axis when thrust pointing requirements do not permit visibility. Transition between these two control criteria shall be smooth. b) Attitude limits during the approach phase, including the effects of site redesignation, shall be 0° to 50° in pitch, and ±30° in bank (coordinates and attitude angles defined later). c) As the phase terminus approached, the LM pitch axis shall approach horizontal and the pitch angle shall approach a value under 15°.

5) Path and velocity objectives: a) In the plan view of the path from the redesignation point to the landing site, the center of curvature shall lie to the same side of the path at all points along the path, i.e., no S turns in the plan view. b) The rate of descent as a function of altitude shall be a smooth curve from a point at 400-ft altitude not exceeding 20 fps, to nominally 0 fps (hover) at an altitude of at least 100 ft. c) The forward velocity as a function of altitude shall be a smooth curve between points at 400-ft altitude not exceeding 70 fps, at 200-ft altitude not exceeding 30 fps, and nominally 0 fps at hover altitude and below.

Guidance Theory

The problem is to guide a spacecraft along a prescribed path using continuous and continuously variable thrust. The solution sought is the total acceleration to be commanded at any point, which includes any gravitational, Coriolis, centripetal, and aerodynamic accelerations as well as thrust acceleration; the thrust acceleration is computed to supply the deficiency between the other accelerations present and the total acceleration required.

The thrusting phases of a mission can be constructed from any number of trajectory segments joined at their extremities. Each trajectory segment is generated using a single guidance equation operating on a single set of guidance coefficients, producing a path along which the position vector and all its derivatives are *nominally* continuous. The approach phase of the lunar landing is considered to be a single trajectory segment even though the terminus may be relocated in flight in response to landing site redesignation commands.

Two members of a family of guidance equations for vehicles with throttleable engines were proposed in Ref. 1. Each equation in the family is capable of generating a trajectory segment on which each component of the position vector is a power series in time. This form of trajectory can satisfy a two-point boundary-value problem in any specified time. The two points are the initial point and the terminal point (terminus) of the trajectory segment, and the boundary conditions are the position vector and any number of derivatives at both points. Considered here is the smaller family of guidance equations capable of satisfying initial conditions only in position and velocity, and any number of terminal conditions.

It is convenient to conceive the trajectory segment as evolving backwards from the terminus. Thus, the segment is completely defined by the position vector and all derivatives at terminus. If we define $\mathbf{R}_F^{(i)}$ as the ith derivative of the position vector at terminus; T as the current time relative to terminus (T is negative prior to reaching terminus); and N, the order of the guidance equation, as the order of the position profile, then the position profile is expressed in the form $\mathbf{R} = \mathbf{R}_F + \ldots + \mathbf{R}_F^{(N)}T^N/N!$. This form exhibits $N + 1$ terminal conditions. $N - 1$ of them may be specified arbitrarily, and the two remaining (normally the two of highest order) must be determined such that the trajectory segment evolves backwards through the current (or initial) position and velocity.

Terminal position and velocity comprise the minimum set of terminal conditions that must be attained to achieve a soft landing. With terminal acceleration and jerk determined to satisfy the initial conditions, the cubic position profile is the minimum form. However, the cubic form is unsatisfactory because redesignation of the landing site during flight can produce a trajectory penetrating the surface enroute to the prescribed terminus. Guidance equations for the cubic and quartic cases were presented in Ref. 1.

The objectives, in terms of velocity, attitude, and visibility profiles, which have been proposed for the lunar landing, define indirectly an ideal landing trajectory. The $N - 1$ arbitrary terminal conditions can be thought of as defining a Maclaurin expansion of the ideal landing trajectory, the expansion being centered about the terminus. Therefore, as a direct consequence of Taylor's theorem, given any set of trajectory specifications which apply in the vicinity of terminus, if there exists an ideal trajectory continuous in the $(N - 1)$th derivative meeting these specifications, then a trajectory can be flown which is an $(N - 2)$th-order expansion of the ideal trajectory by selecting a guidance equation of order N. This is possible even when the terminus is relocated as the trajectory segment is being flown.

To permit relocation (or redesignation) of the terminus in flight, the $N - 1$ arbitrary terminal conditions are fixed in a floating coordinate frame (Fig. 5). When the terminus is moved, the coordinate frame follows. Within this floating frame the position vector and all derivatives at terminus are unchanged, except for the two highest derivatives determined to satisfy the current conditions. Later it will be shown that two components of the second highest derivative can also be constrained by choosing accordingly the orientation of the floating frame and the current time relative to terminus T.

The guidance equation to be derived is an expression for the acceleration to be commanded at any point on the trajectory segment in terms of the $N - 1$ specified terminal conditions and the current velocity and position. The current acceleration, velocity, and position are given by the matrix equation

$$\begin{bmatrix} \mathbf{R}^{(2)} \\ \mathbf{R}^{(1)} \\ \mathbf{R} \end{bmatrix} = \begin{bmatrix} \frac{T^{N-2}}{(N-2)!} & \frac{T^{N-3}}{(N-3)!} & \frac{T^{N-4}}{(N-4)!} & \frac{T^{N-5}}{(N-5)!} & \cdots & 1 & 0 & 0 \\ \frac{T^{N-1}}{(N-1)!} & \frac{T^{N-2}}{(N-2)!} & \frac{T^{N-3}}{(N-3)!} & \frac{T^{N-4}}{(N-4)!} & \cdots & \frac{T}{1!} & 1 & 0 \\ \frac{T^{N}}{N!} & \frac{T^{N-1}}{(N-1)!} & \frac{T^{N-2}}{(N-2)!} & \frac{T^{N-3}}{(N-3)!} & \cdots & \frac{T^2}{2!} & \frac{T}{1!} & 1 \end{bmatrix} [\,\mathbf{R}_F^{(N)}\ \ \mathbf{R}_F^{(N-1)}\ \ \mathbf{R}_F^{(N-2)}\ \ \mathbf{R}_F^{(N-3)}\ \ \ldots\ \ \mathbf{R}_F^{(2)}\ \ \mathbf{R}_F^{(1)}\ \ \mathbf{R}_F\,]^T \tag{1}$$

Note that with the last matrix transposed as indicated, the format of Eq (1) is $[3 \times 3] = [3 \times (N + 1)][(N + 1) \times 3]$. The unknowns are the current (commanded) acceleration $\mathbf{R}^{(2)}$ and the two highest-order derivatives of the position vector at terminus $\mathbf{R}_F^{(N)}$ and $\mathbf{R}_F^{(N-1)}$. The unknowns are separated by transposing terms,

$$\begin{bmatrix} 1 & -\frac{T^{N-2}}{(N-2)!} & -\frac{T^{N-3}}{(N-3)!} \\ 0 & -\frac{T^{N-1}}{(N-1)!} & -\frac{T^{N-2}}{(N-2)!} \\ 0 & -\frac{T^{N}}{N!} & -\frac{T^{N-1}}{(N-1)!} \end{bmatrix} \begin{bmatrix} \mathbf{R}^{(2)} \\ \mathbf{R}_F^{(N)} \\ \mathbf{R}_F^{(N-1)} \end{bmatrix} = \begin{bmatrix} \frac{T^{N-4}}{(N-4)!} & \frac{T^{N-5}}{(N-5)!} & \cdots & 1 & 0 & 0 & 0 & 0 \\ \frac{T^{N-3}}{(N-3)!} & \frac{T^{N-4}}{(N-4)!} & \cdots & \frac{T}{1!} & 1 & 0 & -1 & 0 \\ \frac{T^{N-2}}{(N-2)!} & \frac{T^{N-3}}{(N-3)!} & \cdots & \frac{T^2}{2!} & \frac{T}{1!} & 1 & 0 & -1 \end{bmatrix} [\,\mathbf{R}_F^{(N-2)}\ \ \mathbf{R}_F^{(N-3)}\ \ \ldots\ \ \mathbf{R}_F^{(2)}\ \ \mathbf{R}_F^{(1)}\ \ \mathbf{R}_F\ \ \mathbf{R}^{(1)}\ \ R\,]^T \tag{2}$$

Then the unknowns are found by multiplying through by the inverse of the left-most matrix. This yields

$$\begin{bmatrix} \mathbf{R}^{(2)} \\ \mathbf{R}_F^{(N)} \\ \mathbf{R}_F^{(N-1)} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{(N-4)!} + \frac{(4-N)(N-1)}{(N-2)!}\right) T^{N-4} & \left(\frac{1}{(N-5)!} + \frac{(6-N)(N-1)}{(N-3)!}\right) T^{N-5} & \cdots & \left(\frac{1}{0!} + \frac{(N-4)(N-1)}{2!}\right) T^0 & \left(\frac{(N-2)(N-1)}{1!}\right) T^{-1} & \left(\frac{N(N-1)}{0!}\right) T^{-2} & 2(N-1)T^{-1} & -N(N-1)T^{-2} \\ \frac{1N!}{(N-2)!T^2} & \frac{2N!}{(N-3)!T^3} & \cdots & \frac{(N-3)N!}{2!\,T^{N-2}} & \frac{(N-2)N!}{1!\,T^{N-1}} & \frac{(N-1)N!}{0!\,T^{N}} & \frac{N!}{T^{N-1}} & \frac{-(N-1)N!}{T^{N}} \\ \frac{-2(N-1)!}{(N-2)!T} & \frac{-3(N-1)!}{(N-3)!T^2} & \cdots & \frac{-(N-2)(N-1)!}{2!\,T^{N-3}} & \frac{-(N-1)(N-1)!}{1!\,T^{N-2}} & \frac{-N!}{0!\,T^{N-1}} & \frac{-(N-1)!}{T^{N-2}} & \frac{N!}{T^{N-1}} \end{bmatrix} [\,\mathbf{R}_F^{(N-2)}\ \ \mathbf{R}_F^{(N-3)}\ \ \ldots\ \ \mathbf{R}_F^{(2)}\ \ \mathbf{R}_F^{(1)}\ \ \mathbf{R}_F\ \ \mathbf{R}^{(1)}\ \ \mathbf{R}\,]^T \tag{3}$$

The expression for $\mathbf{R}^{(2)}$ [the first line of Eq. (3)] is the required guidance equation.

The guidance equation [the first line of Eq. (3)] will produce a trajectory that satisfies terminal conditions on $\mathbf{R}_F^{(N-2)}$ through $\mathbf{R}_F$. We now show how two remaining degrees of freedom are expended to satisfy two additional terminal constraints on $\mathbf{R}_F^{(N-1)}$. Nothing preceding depended upon any specific orientation of the floating coordinate frame. The following does, and this will be considered first.

Because the landing site is the direct result of any redesignation commands, one axis (i of Fig. 5) is defined along the local vertical of the landing site. Normally, the terminal conditions $\mathbf{R}_F^{(N-2)}$ through $\mathbf{R}_F$ are specified to lie in the ki plane, which makes ki the plane of the trajectory at terminus, defined by the terminal position and terminal velocity vectors relative to the center of the attracting body. Because the landing site can be approached from any direction, the orientation of the frame about the vertical (i) is arbitrary, and is determined such that $\mathbf{R}_F^{(N-1)}$ also lies in the ki plane.† This leaves only $\mathbf{R}_F^{(N)}$ outside the plane of the trajectory at terminus.

Thus, granted the required orientation of the floating frame, the j components $R_{Fj}^{(N-1)}$ through R_{Fj} must all be zero. Rewriting the jth scalar equation contained in the third line of Eq. (3) for this case,

$$R_j N!/T^{N-1} - R_j^{(1)}(N-1)!/T^{N-2} = 0 \tag{4}$$

Multiplying through by $T^{N-1}/N!$ yields

$$R_j - R_j^{(1)}\, T/N = 0 \tag{5}$$

which is equivalent to

$$(\mathbf{R} - \mathbf{R}^{(1)}T/N)_j = 0 \tag{6}$$

Equation (6) implies that the j axis in Fig. (5) is perpendicular to the vector within parentheses. Therefore, the unit vectors of the floating coordinate frame are

$$\mathbf{i} = \mathrm{unit}(\mathbf{L}) \tag{7}$$

$$\mathbf{j} = \mathrm{unit}[\mathbf{L} \times (\mathbf{R} - \mathbf{R}^{(1)}T/N)] \tag{8}$$

$$\mathbf{k} = \mathbf{i} \times \mathbf{j} \tag{9}$$

where $\mathbf{L}$ is the landing site vector, and $\mathbf{L}$, $\mathbf{R}$, and $\mathbf{R}^{(1)}$ may now be expressed in any frame that originates at the center of and rotates with the attracting body.

The preceding terminal conditions will be satisfied for arbitrary time T. Therefore, T is determined to satisfy one additional boundary constraint, which we choose to impose on $R_{Fk}^{(N-1)}$, the horizontal component of $\mathbf{R}_F^{(N-1)}$. Thus, T is found as one root of a polynomial derived from the scalar expression for $R_{Fk}^{(N-1)}$ contained in the third line of Eq. (3). Separating out, this expression yields

$$\begin{aligned} R_{Fk}^{(N-1)} = {} & \frac{-2(N-1)!}{(N-2)!T} R_{Fk}^{(N-2)} + \frac{-3(N-1)!}{(N-3)!T^2} \times \\ & R_{Fk}^{(N-3)} + \ldots + \frac{-(N-2)(N-1)!}{2!T^{N-3}} R_{Fk}^{(2)} + \\ & \frac{-(N-1)(N-1)!}{1!T^{N-2}} R_{Fk}^{(1)} + \frac{-N!}{0!T^{N-1}} R_{Fk} + \\ & \frac{-(N-1)!}{T^{N-2}} R_k^{(1)} + \frac{N!}{T^{N-1}} R_k \end{aligned} \tag{10}$$

Transposing the right side, multiplying through by T^{N-1}, and combining terms yields the polynomial

† This problem was first solved by a numerical iterative approach. A closed-form solution for the quartic case (by a different approach than presented here) was first suggested, in informal correspondences, by T. E. Moore of the NASA Manned Spacecraft Center, Houston, Texas.

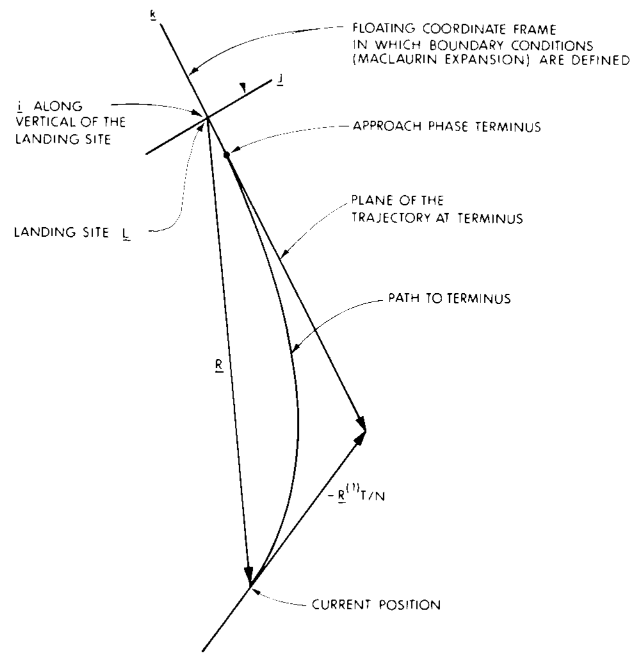

Fig. 5 Plan view showing floating coordinate frame oriented per terminal trajectory plane. (Underscored symbols in figures indicate vectors, although, in the text, vectors are denoted by boldface type.)

$$\begin{aligned} & R_{Fk}^{(N-1)}T^{N-1} + \frac{2(N-1)!}{(N-2)!} R_{Fk}^{(N-2)}T^{N-2} + \\ & \frac{3(N-1)!}{(N-3)!} R_{Fk}^{(N-3)}T^{N-3} + \ldots + \frac{(N-2)(N-1)!}{2!} \times \\ & R_{Fk}^{(2)}T^2 + \left[\frac{(N-1)(N-1)!}{1!} R_{Fk}^{(1)} + \right. \\ & \left. (N-1)!R_k^{(1)}\right] T + N!(R_{Fk} - R_k) = 0 \end{aligned} \tag{11}$$

One root of this polynomial is the required time T.

The solution for the orientation of the floating coordinate frame, Eqs. (7–9), is not independent of the solution for time [Eq. (11)]. The interdependence can be ignored in practice because guidance equations are normally processed repetitively, which produces rapid convergence without additional iterations on these targeting equations.

Definitions of Coordinates and Attitude Angles

Three coordinate systems are carried by the guidance computer. They include the effects of current and accumulated sensor errors. For example, the inertial coordinates actually rotate as the gyros drift. Figure 6 pertains to the following coordinate definitions:

Body coordinates: These are the generally accepted LM coordinates. The X axis is in the direction of the nominal thrust vector, the Z axis is in the direction forward from the design eye, and the Y axis completes a right-hand triad XYZ. Variables in body coordinates are tagged B. The unit vectors of the body frame with respect to the inertial frame are, along the X,Y,Z axes, the row vectors $\mathbf{CB}_0$, $\mathbf{CB}_3$, $\mathbf{CB}_6$ of the matrix $\hat{C}B$. (A caret over a symbol indicates a matrix.)

Inertial coordinates: These are the inertial measurement unit coordinates. The origin is at the center of the moon, the X axis pierces the nominal (unredesignated) landing site at the nominal landing time, the Z axis is parallel to the orbital plane of the Command Module and points forward,

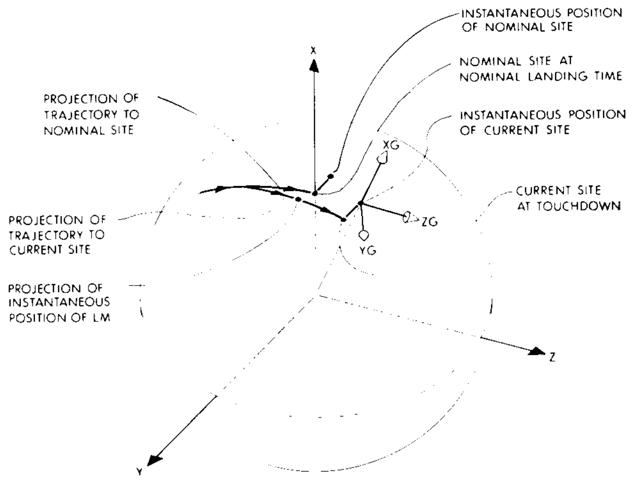

Fig. 6 Coordinates definitions viewed by a stationary observer.

and the Y axis completes a right-hand triad XYZ. Thus, if the nominal site is in the plane of the Command Module orbit, and if the LM lands at the nominal site at the nominal time and in an erect and nominal attitude, the inertial and body coordinates will be parallel at the instant of landing. Variables in inertial coordinates are untagged.

Guidance coordinates: The origin coincides continuously with the current landing site (the frame rotates with the moon). The X axis is vertical, the Z axis lies in the plane of the trajectory relative to the moon at phase terminus and points forward, and the Y axis completes a right-hand triad XYZ. Thus, the origin and orientation of the guidance frame are altered each time the landing site is redesignated. Variables in guidance coordinates are tagged G. The unit vectors of the guidance frame with respect to the inertial frame are, along the X,Y,Z axes, the row vectors $\mathbf{CG}_0$, $\mathbf{CG}_3$, $\mathbf{CG}_6$ of the matrix $\hat{C}G$. The attitude angles are a set of Euler angles defined in Table 1.

Steering and Guidance Computations

The order of the guidance equation is chosen as a design compromise. The higher the order, the more closely a trajectory to a redesignated landing site adheres to the ideal, but also the more severely the spacecraft responds to site redesignation and noise introduced into the state vector by the landing radar. For example, it is immediately apparent from Fig. 5 that the infinite-order guidance equation responds to site redesignation by commanding infinite acceleration in order to produce a straight line trajectory (in the plan view) to the redesignated site. Quartic has been selected because it is the minimum order capable of meeting the design objectives for guidance logic, etc. when the redesignated site is within the corresponding landing footprint (Fig. 4).

Figure 7 is a block diagram of the guidance system for the approach phase. Although this is a digital system, it is also a closed-loop or feedback system. The block diagram format is chosen because it seems to display the essential feedback characteristics better than a digital logic diagram. Of course, the operations represented by the blocks are performed sequentially rather than continuously, and there are time delays in the feedback information. The block diagram is divided into four sections: 1) the manual steering section simply provides the targeting equations with a landing site; 2) the targeting equations determine the guidance coordinate unit vectors, the current state in guidance coordinates, and the current time relative to terminus; 3) the guidance equations yield the throttle commands and the attitude error signals; and 4) the guidance execution section performs the remaining functions required to carry out the guidance commands and to produce the input data for the guidance system. The sections of the block diagram are processed successively from top to bottom, and iteratively with a time interval of 2 sec. Of course, the real variables change during the processing time. Therefore, as nearly simultaneously as possible, the clock and accelerometers are read and the readings stored. A set of variables consistent with this time is computed and used throughout one pass through the guidance equations. One exception to this is necessary in the targeting equations section. Because there is a closed loop in this section and each operation in the loop is performed only once during one pass, one operation must use outdated data. In Fig. 7 the relative time used to compute the guidance coordinate unit vectors is obtained by updating the value from the previous pass, and does not reflect any intervening site redesignations. Simulation of this loop has revealed no resulting detrimental effects.

Manual Steering

The manual steering equations must produce a current landing site in inertial coordinates and a display number indicating the site. The new (current) landing site is produced by the motion of the moon relative to the inertial coordinates, and by the steering commands. The landing site in inertial coordinates and the corresponding time are maintained as a consistent set. To determine the site on the current pass, the landing site of the previous pass is rotated in the inertial coordinates by the angle traversed by the moon since the previous pass. Then the site is corrected according to any manual steering commands received during the interval, and stored along with the current time for future reference.

There is no way to synchronize the manual commands with passes through the guidance equations. Consequently, it is necessary to interrupt the computer when a steering command is received and increment or decrement the azimuth or elevation command count. On the coming pass, the command counts are used by the landing site perturbation equations (Fig. 7) and then reset to zero. The angular increments to be imparted to the landing site have been tentatively selected as $\frac{1}{2}°$ in body elevation per fore-aft manipulation of the controller, and 2° in body azimuth per left-right manipulation of the controller.

Given the landing site updated for lunar rotation, the landing site perturbation equations first determine a point near the surface displaced by the proper angle from the given site, (see Fig. 8). Then the new landing site is placed directly beneath this point. Thus, a vector to the displaced point near the surface is computed as

$$\mathbf{VCT} = \hat{C}BT \begin{bmatrix} UNLRB_0 + NCEL\ ELPRT\ UNLRB_2 \\ UNLRB_1 + NCAZ\ AZPRT \\ UNLRB_2 - NCEL\ ELPRT\ UNLRB_0 \end{bmatrix} \qquad (12)$$

where **UNLRB** is a unit vector in body coordinates along the line of sight to the site updated for lunar rotation, the X,Y,Z components are numbered 0,1,2, $NCEL$ is the elevation command count, $ELPRT$ is the elevation angular increment in radians, $NCAZ$ and $AZPRT$ are the azimuth count and azimuth increment, and $\hat{C}BT$, the transpose of $\hat{C}B$, is the transformation to inertial coordinates. Note that the vector in brackets is **UNLRB** rotated by angular components $NCEL$

Table 1 Definition of attitude angles

Attitude angle	Euler angle	Rotation about
Heading	Inner	X axis
Pitch	Middle	Displaced Y axis
Bank	Outer	Displaced Z axis

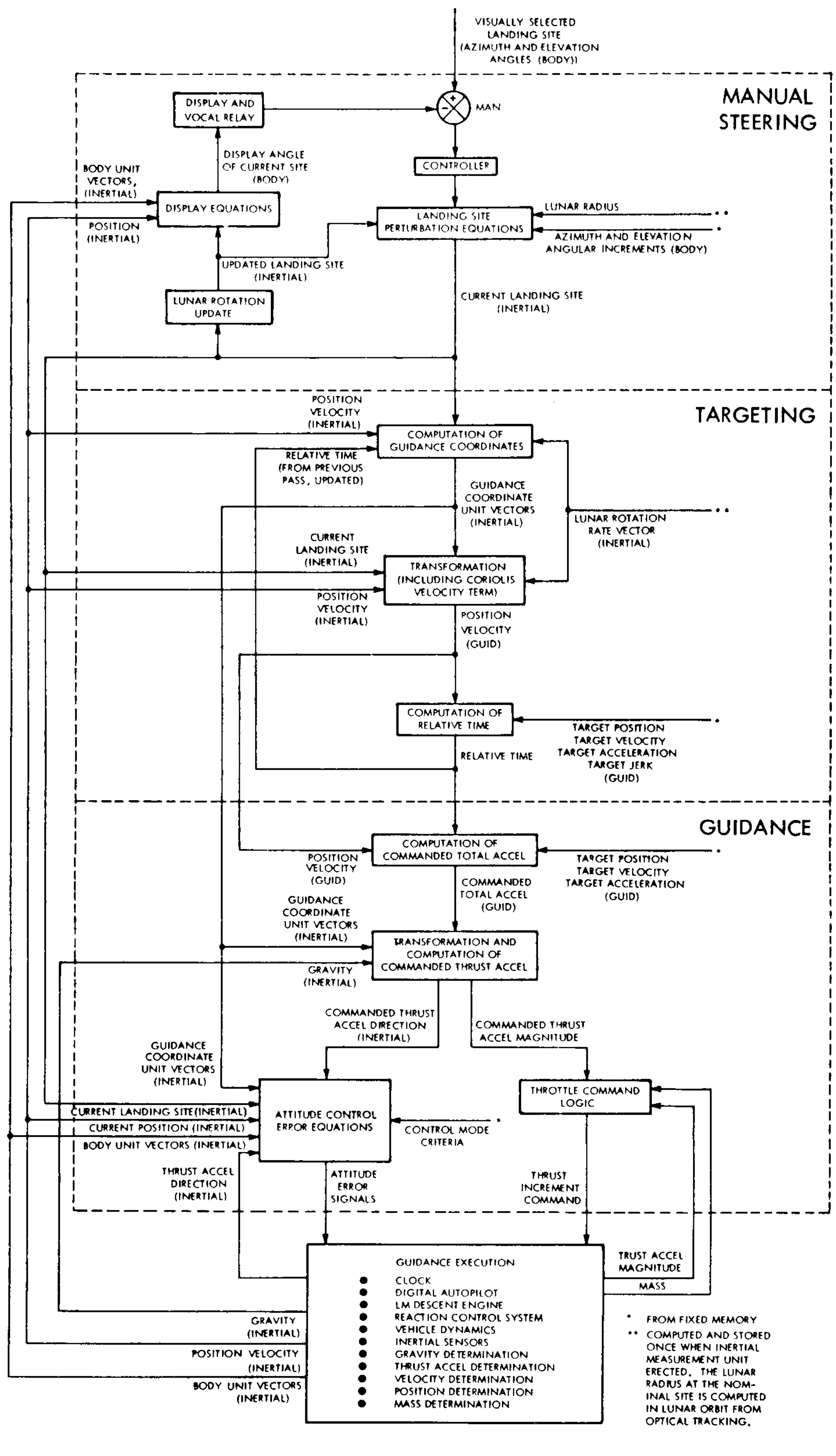

Fig. 7 Approach phase guidance in detail.

$ELPRT$ and $NCAZ\ AZPRT$, using small-angle approximations. The new landing site is calculated as

$$\mathbf{L} = RM \text{ unit}[\mathbf{R} + \mathbf{VCT}(L_0 - R_0)/VCT_0] \qquad (13)$$

where RM is the lunar radius, and $\mathbf{R}$ is the current LM position.

The number to display telling the commander where to look to see the new landing site is

$$DISPLAY = \arcsin(-UNLRB_0) \qquad (14)$$

where **UNLRB** is recomputed for the new site.

Targeting

With the landing site determined, the targeting equations must compute the guidance coordinate unit vectors, the current state in this rotating frame, and the current time relative to terminus. (Note that throughout this paper, time T is defined relative to phase terminus and, as such, is the negative of time-to-go in the phase.) The target parameters are the terminal boundary conditions specified for the mission phase. They are called target or desired parameters because practical limitations preclude achieving terminal boundary conditions exactly. The target parameters

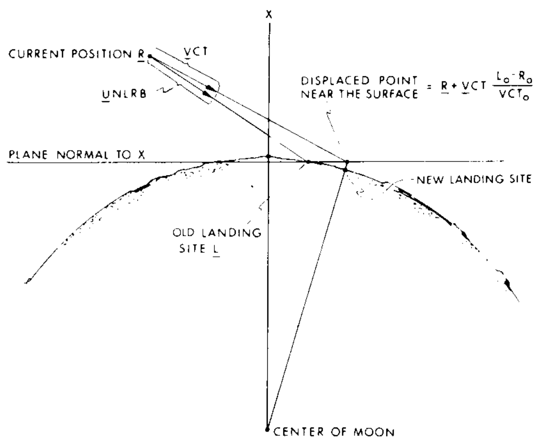

Fig. 8 Landing site perturbation geometry.

are denoted by the tag D (for desired) preceding the tag denoting the coordinate frame, e.g., the target acceleration vector in guidance coordinates is denoted **ADG**. The design objectives, guidance theory, and coordinates definitions described previously are the basis for the targeting approach summarized as follows:

1) The quartic guidance equation is selected. That is, the profiles followed by components of the position vector are quartic functions of time, and the commanded acceleration profiles are quadratic.

2) The target vectors which can be achieved in any time and with any coordinate frame orientation are the terminal position, velocity, and acceleration.

3) All target parameters are defined in the guidance coordinate frame, tagged G, whose origin coincides continuously with the landing site.

4) The frame is oriented about the vertical such that the YG component of the jerk vector will achieve the target value of zero (the terminal jerk vector achieved will lie in the ZG XG plane).

5) Time relative to terminus T is computed such that the ZG component of the jerk vector will achieve a specified target value.

Because the target parameters are invariant in guidance coordinates, erecting the guidance frame constitutes defining the inertial target parameters. Several desirable features result from this targeting approach: 1) with the terminal acceleration vector and the landing site vector lying in the ZG XG plane, and attitude control such that these same vectors also lie in the ZB XB plane, these planes are coincident at terminus, therefore, the terminal bank angle is zero; 2) with the terminal jerk vector lying in the ZB XB plane, the body angular rate vector is normal to this plane at terminus, and the terminal attitude rates are therefore zero except for pitch, and 3) because the ZG components of the tangent velocity, acceleration, and jerk are alternate in sign, i.e., + − +, there can be no S turns in the plan view of the path to a redesignated landing site.

On each pass the current time relative to terminus is updated by $T = T + DT$, where DT is the elapsed time since the previous pass. Using this, the new guidance coordinate unit vectors are computed as suggested by Fig. (5) and Eqs. (7–9). Thus, using $N = 4$,

$$\hat{C}G = \begin{bmatrix} \text{unit }(\mathbf{L}) \\ \text{unit}\{\mathbf{L} \times [\mathbf{R} - (\mathbf{V} - \mathbf{WM} \times \mathbf{R})T/4]\} \\ \mathbf{CG}_0 \times \mathbf{CG}_3 \end{bmatrix} \quad (15)$$

where **WM** is the angular rate vector of the moon, and $\mathbf{CG}_0$, $\mathbf{CG}_3$, $\mathbf{CG}_6$ are the row vectors of $\hat{C}G$. The term $-\mathbf{WM} \times \mathbf{R}$ must be included to obtain the velocity relative to the moon.

The current position and velocity in guidance coordinates are

$$\mathbf{RG} = \hat{C}G(\mathbf{R} - \mathbf{L}) \qquad \mathbf{VG} = \hat{C}G(\mathbf{V} - \mathbf{WM} \times \mathbf{R}) \quad (16)$$

A refined value of T is computed using Newton's method with Eq. (11) for $N = 4$. First, a convergence criterion is computed as $DTCRIT = |T/128|$. Then the change in T such as to satisfy Eq. (11) for this case is computed as

$$DT = -\frac{JDG_2T^3 + 6ADG_2T^2 + (18VDG_2 + 6VG_2)T + 24(RDG_2 - RG_2)}{3JDG_2T^2 + 12ADG_2T + 18VDG_2 + 6VG_2} \quad (17)$$

where JDG_2, ADG_2, VDG_2, and RDG_2 are the ZG target values for jerk, acceleration, velocity, and position. Note that the numerator of Eq. (17) comes directly from Eq. (11), and the denominator is the derivative of the numerator with respect to T. The refined value of T is computed as $T = T + DT$. If the magnitude of DT exceeds the convergence criterion $DTCRIT$, control is returned to Eq. (17) for another iteration.

Guidance Equations

With all target parameters determined, the guidance equations must produce throttle commands and attitude error signals.

Throttle commands

The commanded total acceleration (includes gravity) is calculated using the current state and target parameters in guidance coordinates, and a guidance equation obtained from the first line of Eq. (3) for $N = 4$. Thus,

$$\mathbf{ACG} = \mathbf{ADG} + 6(\mathbf{VDG} + \mathbf{VG})/T + 12(\mathbf{RDG} - \mathbf{RG})/T^2 \quad (18)$$

The commanded thrust acceleration in inertial coordinates is obtained using $\hat{C}GT$, the transpose of the transformation matrix $\hat{C}G$, and subtracting the gravitational acceleration already present. Thus,

$$\mathbf{AFC} = \hat{C}GT\,\mathbf{ACG} - \mathbf{G} \quad (19)$$

The Coriolis and centripetal accelerations due to rotation of the guidance frame at lunar rate are negligible and ignored. The thrust increment (throttle) command is

$$DFC = M(AFC - AF) \quad (20)$$

where M is the current mass, AFC is the magnitude of **AFC**,

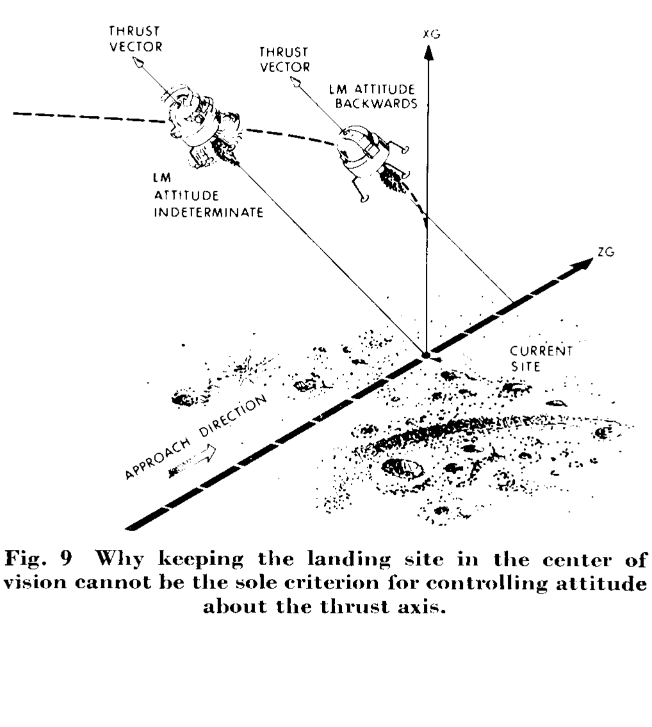

Fig. 9 Why keeping the landing site in the center of vision cannot be the sole criterion for controlling attitude about the thrust axis.

and AF is the magnitude of the current measured thrust acceleration.

At some time near the end of the phase it becomes numerically unfeasible to use Eq. (18) because T approaches zero. After this, the commanded acceleration is calculated as a linear function of time from the value last computed by Eq. (18) to the target value. Thus, immediately after Eq. (18) is processed for the last time, the linear acceleration coefficients are stored as

$$\mathbf{JLING} = (\mathbf{ACG} - \mathbf{ADG})/T \tag{21}$$

On subsequent passes the commanded acceleration is computed as

$$\mathbf{ACG} = \mathbf{ADG} + \mathbf{JLING}\, T \tag{22}$$

During linear acceleration guidance, erection of the guidance frame [Eq. (15)] is omitted. The previously computed unit vectors are retained for all succeeding requirements.

Attitude control error equations

The objectives of the attitude control error equations are 1) to bring the commanded and actual thrust acceleration vectors into collinearity and 2) to control the attitude about the thrust axis such that the current landing site is kept in the center of vision (in the LM ZB XB plane). Achieving 1) is simple, but there are pitfalls in attempting to achieve 2). These pitfalls are illustrated in Fig. 9.

Attitude control error equations are presented which produce a satisfactory attitude at all times during the descent. For control about the thrust axis (actually the error signals are used for control about the XB axis), two control modes are used, with a smooth transition between them. Figure 10 shows the geometry pertinent to generating error signals about the XB axis. If we define **UNLR** as a unit vector along the line of sight to the landing site, **UNLR** = unit (**L** − **R**), then the normal to the plane formed by the line of sight vector and the XB axis is

$$\mathbf{NORMLRX} = \mathbf{UNLR} \times \mathbf{CB}_0 \tag{23}$$

Note that the length of **NORMLRX** is the sine of the look angle, Fig. 1, and the direction is the direction the YB axis must point for the landing site to lie in the ZB XB plane. Therefore, the YB axis is aligned with **NORMLRX** when doing so would produce an acceptable attitude; otherwise, the angle between the YB and YG axes is minimized, facing the LM forward in the direction of motion. A smooth transition is made between these two control modes.

Scrutiny of Fig. 10 indicates that aligning the YB axis with **NORMLRX** produces a normal forward-facing attitude when **NORMLRX** points in the direction of the YG axis; it produces a rear-facing attitude when **NORMLRX** points opposite to the YG axis; and it produces a side-facing attitude when **NORMLRX** is normal to the YG axis, which does happen in nonplanar landings. Therefore, the projection of **NORMLRX** on the YG axis is used as the criterion for

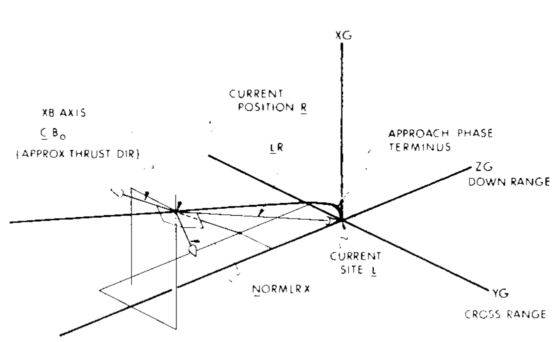

Fig. 10 Geometry pertinent to generation of attitude error signals about the XB axis.

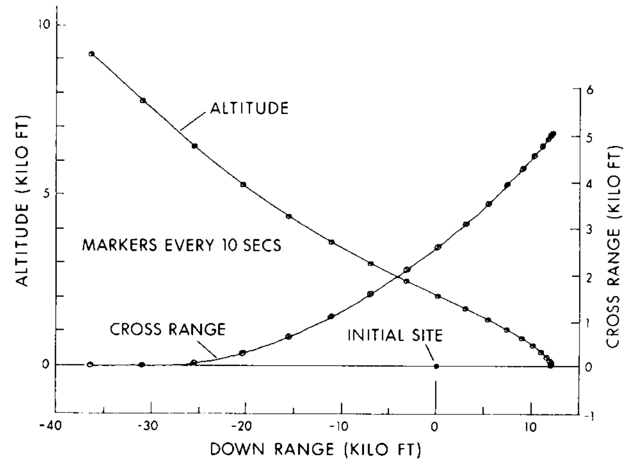

Fig. 11 Elevation and plan views in lunar-fixed coordinates.

selecting control mode. This criterion works even when the LM is below the YG ZG plane, as it is during most of the preceding braking phase.

The projection of **NORMLRX** on the YG axis is $PROJ$ = **NORMLRX**·$\mathbf{CG}_3$. Because the magnitude of **NORMLRX** is the sine of the look angle (Fig. 1), $PROJ$ is the sine of the look angle multiplied by the cosine of the angle between **NORMLRX** and the YG axis. If $PROJ$ is algebraically greater than the sine of 25° ($PROJMAX$), then aligning the YB axis with **NORMLRX** will produce an essentially forward-facing attitude *and* the landing site will be visible. In this case, the error signal for control about the XB axis is computed as

$$ERXL = -[\text{unit}(\mathbf{NORMLRX})]\cdot\mathbf{CB}_6 \tag{24}$$

where $\mathbf{CB}_6$ is a unit vector in the ZB direction. If $PROJ$ is algebraically less than the sine of 15° ($PROJMIN$), then aligning the YB axis with **NORMLRX** either would not produce a forward-facing attitude *or* the landing site would not be visible, or both. In this case the angle between the YB and YG axes must be minimized, and the error signal is computed as

$$ERXG = -\mathbf{CG}_3\cdot\mathbf{CB}_6 \tag{25}$$

If $PROJ$ is between $PROJMAX$ and $PROJMIN$, then both $ERXL$ and $ERXG$ are computed, and combined to yield the error signal

$$ERX = ERXL(PROJ - PROJMIN)/(PROJMAX - PROJMIN) + ERXG(PROJ - PROJMAX)/(PROJMIN - PROJMAX) \tag{26}$$

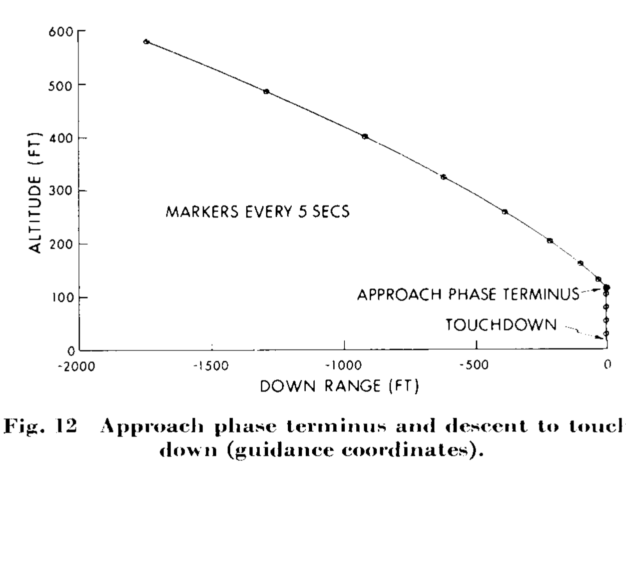

Fig. 12 Approach phase terminus and descent to touchdown (guidance coordinates).

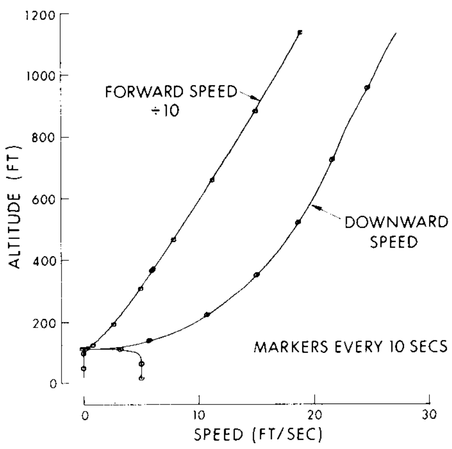

Fig. 13 Forward and downward speeds vs altitude.

The error signals about the *YB* and *ZB* axes, required to correct the pointing of the thrust vector, are calculated as

$$ERY = \mathbf{UNAFC} \times \mathbf{UNAF} \cdot \mathbf{CB}_3 \tag{27}$$

$$ERZ = \mathbf{UNAFC} \times \mathbf{UNAF} \cdot \mathbf{CB}_6 \tag{28}$$

where **UNAFC** and **UNAF** are unit vectors in the direction of the commanded thrust acceleration **AFC** and the measured thrust acceleration **AF**.

Sample Trajectory

Figures 11–15 present a sample trajectory that could be used for a lunar landing. This trajectory is presented for illustration only and is not representative of any planned mission. The curves were drawn by an $X - Y$ plotter driven by a digital computer simulating a landing. The coordinates

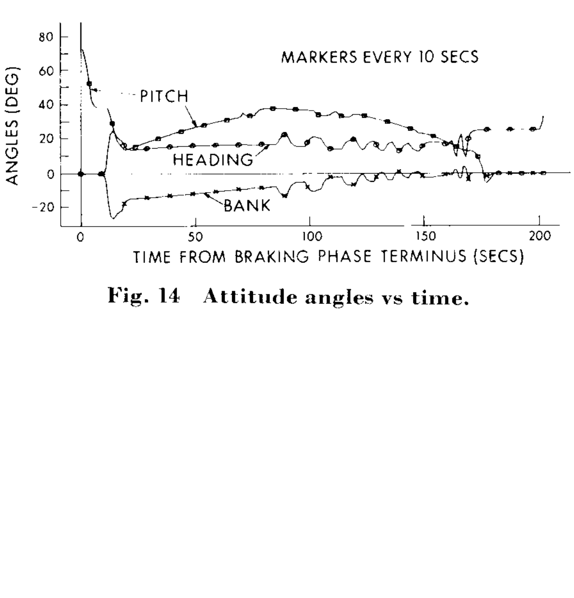

Fig. 14 Attitude angles vs time.

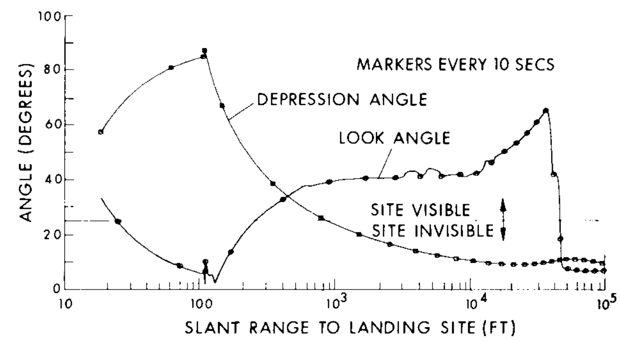

Fig. 15 Look angle and depression to designated site.

of Fig. 11 are a lunar-fixed set that coincides with the inertial set at the nominal landing time. This sample trajectory landed on the left edge of the landing footprint shown in Fig. 4; the designated landing site was 12,000 ft forward and 5,000 ft to the left of the unredesignated site. The manual steering commands which produced this trajectory were generated by a digital simulation of the human commander. The simulated human visually acquired the landing site at 5 sec into the approach phase. At 6 sec he decided to issue five commands to the left and seven commands forward. He issued the azimuth commands first and then the elevation commands at the rate of one command every 0.4 sec. He then waited for the attitude transient to decay and made a new decision to issue one command to the right and four more commands forward. Convergence required one further command, backward. After 16 sec, commands were necessary only to refine convergence and were issued infrequently at random times during the remainder of the approach phase. A total of 39 commands was issued.

The irregularities in the curves of attitude angles, Fig. 14 are produced by steering commands. It should be noted that even with manual steering, as the LM approaches the terminus, the bank angle approaches zero, and therefore the horizon approaches a horizontal attitude in the window.

References

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[2] Kriegsman, B. A. and Gustafson, D. E., "Powered Landing Guidance-and-Navigation Systems for LM," *Advances in Astronautical Sciences*, Vol. 24.